

Name: _____

5.5 Escape Velocity

Recall from last class playing with Angry Birds...

With enough velocity, we're able to send a bird into orbit. What happens if we continue to increase the velocity of the bird we shoot outwards?

That means there must be a minimum required velocity for an object to leave the gravitational pull of a planet/star at its surface. Let's analyze this from an energy perspective:

Kinetic energy \rightarrow Gravitational potential energy

Initial: this is object launching off the surface of the planet/star with velocity v_i , which has a radius of R_i

Final: the object completely leaves the gravitational influence of the planet/star so $R_f \rightarrow \infty$. For the minimum amount of required velocity for this object, we only need enough kinetic energy to this object to escape to infinity. Therefore, we don't need any kinetic energy left at infinity.

Conservation of energy states that

$$\Delta E_k = -\Delta E_p$$

$$E_{K_f} = 0 \text{ J}$$

$$E_i = E_f$$

$$E_{K_i} + E_{P_i} = E_{K_f} + E_{P_f}$$

$$\frac{1}{2}mv_i^2 + \left(-\frac{GMm}{R_i}\right) = \frac{1}{2}mv_f^2 + \left(-\frac{GMm}{R_f}\right)$$

Since $v_f \rightarrow 0 \text{ m/s}$ and $R_f \rightarrow \infty$ when escaped,

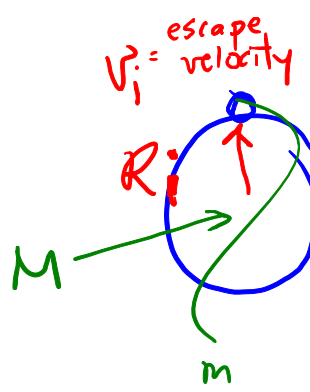
$$\frac{1}{2}mv_i^2 - \frac{GMm}{R_i} = 0$$

$$\Rightarrow \frac{1}{2}v_i^2 = \frac{GM}{R_i} \times 2 \Rightarrow v_i = \sqrt{\frac{2GM}{R_i}}$$

$$-\frac{1}{2}mv_i^2 = -\frac{GMm}{R_i} \Rightarrow \text{solving for } v_i$$

$$v_i = \sqrt{\frac{2GM}{R_i}}$$

notice that v_{escape} does NOT depend on the object's mass



Ex. 1: At what speed do you need to throw a 1.0 kg rock in order for it to leave the Earth's gravitational pull? (ANS: 11000m/s)

$$v_i = \sqrt{\frac{2GM}{R_i}} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}}$$

$$= 11000 \text{ m/s}$$

$E_K = \frac{1}{2}mv_i^2$
 but E_{K_i} does depend on mass.

Does the mass of the rock matter?
 How much kinetic energy would it need?

$$E_K = \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}(1.0 \text{ kg})(1.1 \times 10^4 \text{ m/s})^2$$

$$= 6.3 \times 10^7 \text{ J}$$

HW: Worksheet 5.5 #1, 4, 5, 6