Name: ________________________________ Phys 12

5.4 Gravitational Potential Energy in Circular Motion

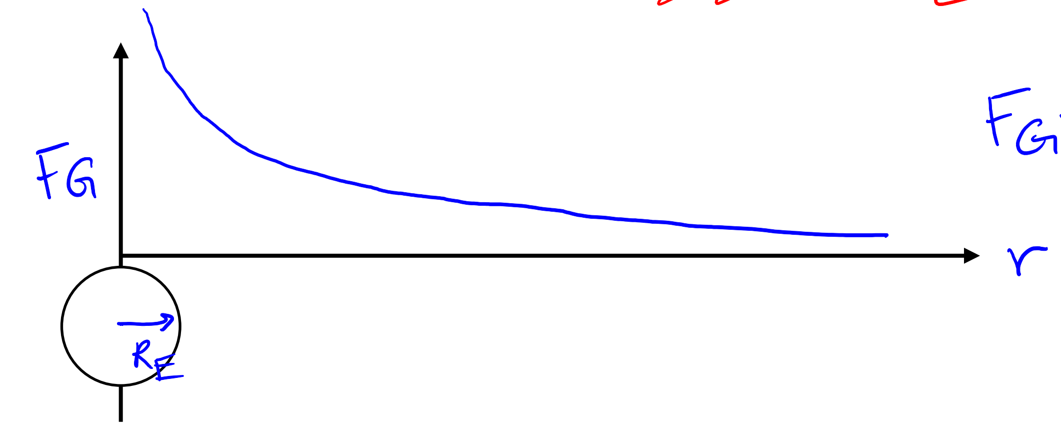
Back in the day… $E_p = \boxed{mg}h$ F_g

Where *h* is relative to the ground (usually), so you'd have ZERO potential energy on the ground.

What happens when the force due to gravity ~~________________~~ changes the higher up you go?

$F_G \downarrow$

$$F_G = \frac{GMm}{r^2}$$



Since the force due to gravity is changing depending how far you are, we cannot use E_p = mgh.

Instead, we use

$$E_p = -\frac{GMm}{r}$$

Where G = Gravitational Constant $(6.67\times10^{-11}\ N\cdot m^2/kg^2)$

M = Mass of "bigger" object (kg)

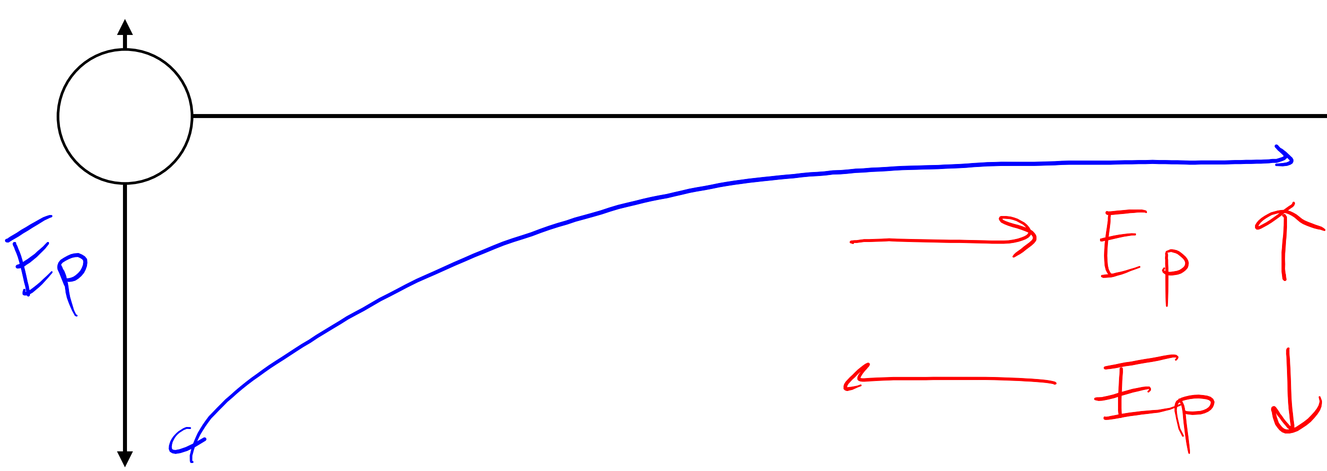
m = Point mass, mass of "smaller" object (kg)

r = distance between the centre of masses of M and m (metres)

Instead of the reference point being the surface of M, physicists chose the reference point to be ZERO at infinity. Thus, the potential energy of m will be compared to infinity. (Why you ask? Calculus…) Why is the equation negative? …. also because of Calculus. However, we can see why the negative sign makes sense by analyzing how potential energy should change going away from M and moving closer to M:

$E_p \rightarrow 0\,J$

$R \rightarrow \infty$



$\longrightarrow E_p \uparrow$

$\longleftarrow E_p \downarrow$

Ex. 1: A 2500 kg satellite is in orbit 3.60×10^7 m **above** the Earth's surface. What is the gravitational potential energy of the satellite due to the gravitational force due to the Earth? (ANS: -2.4×10^{10} J)

$$E_p = -\frac{GMm}{r} = -\frac{6.67\times10^{-11}(5.98\times10^{24})(2500\,kg)}{(3.6\times10^{7}\,m + 6.38\times10^{6}\,m)}$$

$$= \boxed{-2.4\times10^{10}\,J}$$

RE

HW: Worksheet 5.4 (all)

*note $E_T = \frac{1}{2}E_p$ ONLY for orbits $\quad E_k = -\frac{1}{2}E_p$ yes!

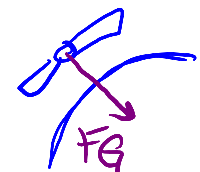
Name: ______ONLY for orbits______ Phys 12

Ex. 2: What is the **total** energy of the satellite in the last question? (Hint: does the satellite have kinetic energy?) (ANS: -1.2 x 10^{10} J)

$$E_T = E_p + E_k$$
$$= -\frac{GMm}{r} + \frac{1}{2}mv^2$$
$$= -\frac{GMm}{r} + \underbrace{\frac{1}{2}m\left(\frac{GM}{r}\right)}_{E_K}$$
$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$
$$= -\frac{1}{2}\left(\frac{6.67\times10^{-11}(5.98\times10^{24})(2500)}{3.6\times10^7\,m + 6.38\times10^6\,m}\right) = \boxed{-1.2\times10^{10}\,J}$$

$$F_{net} = F_G = ma_c$$
$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$
$$v^2 = \frac{GM}{r}$$

note this is ½ of Ex. 1!

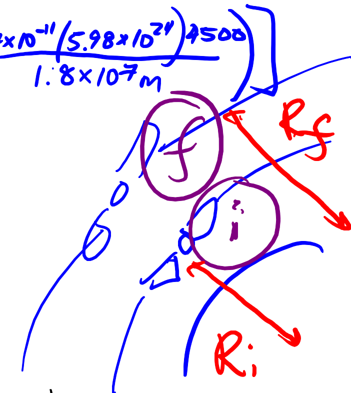


Ex. 3: How much work is required to move a 4500 kg Earth satellite from an orbital radius of 1.8x10^7 m to an orbital radius of 4.2x10^7 m? (ANS: 2.8 x 10^{10} J)

for E_T relationships, refer to Ex. 2.

we know: $E_T = \frac{1}{2}E_p$ for orbits

$$W = E_{T_f} - E_{T_i}$$
$$= \frac{1}{2}E_{p_f} - \frac{1}{2}E_{p_i}$$
$$= \frac{1}{2}\left[\left(-\frac{GMm}{R_f}\right) - \left(-\frac{GMm}{R_i}\right)\right]$$
$$= \frac{1}{2}\left[\left(-\frac{6.67\times10^{-11}(5.98\times10^{24})4500}{4.2\times10^7\,m}\right) + \left(+\frac{6.67\times10^{-11}(5.98\times10^{24})4500}{1.8\times10^7\,m}\right)\right]$$
$$= \boxed{2.8\times10^{10}\,J}$$



Worksheet 5.4 Gravitational Potential Energy in Circular Motion *(complete on separate sheet of paper)*

1) What is the gravitational potential energy (relative to infinite) of a 5.00x10^3 kg satellite that is in orbit with a radius of 9.90x10^6 m around the Earth? (-2.0x10^{11} J)

2) How much work is done **against gravity** in lifting the satellite in problem #1 from Earth's surface to its orbital height? (1.11x10^{11} J)

3) What is the total energy of the orbiting satellite in problem #1? (-1.0 x 10^{11} J)

4) How much work is done in lifting the satellite in problem #1 **from rest** on the Earth's surface to its orbit at 9.90x10^6 m around the Earth? (2.1 x 10^{11} J)

5) A 1750 kg meteorite is 15000 m above the surface of the moon, heading directly towards the moon at 1.00x10^3 m/s. What is its speed on impact? (1.02x10^3 m/s)

6) What is the gravitational potential energy of a 10.0 kg object when it is sitting on Earth's surface? (-6.25x10^8 J)

7) A 12500 kg satellite is in Earth orbit at an altitude of 3.60x10^6 m. What is its **total** energy? (-2.50x10^{11} J)

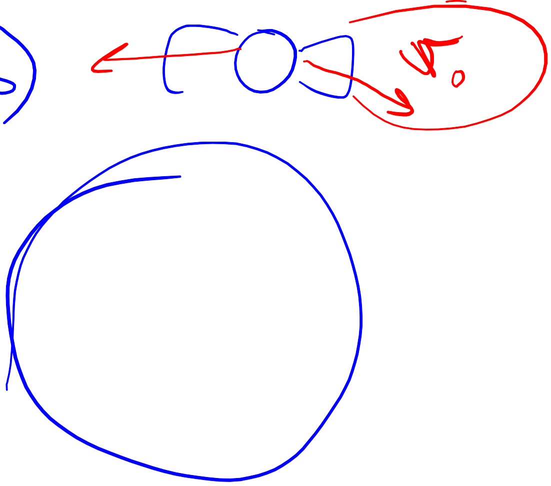
ejected backwards

8) The International Space Station ~~drops~~ a 250 kg waste shuttle from an altitude of 3.50x10^5 m. At what speed would it impact Earth if there were no air friction? (Assume it starts at rest) (ANS: 2600 m/s)

9) A 2.35x10^{16} kg asteroid falls (released from rest) towards the Earth from a really, really, REALLY far way away. What is its velocity right before it impacts with the Earth? (ANS: 1.1 x 10^4 m/s)

HW: Worksheet 5.4 (all)

8)



$v_i = 0\,m/s$

$v_f = ?$

$R_i = 3.50 \times 10^5\,m + 6.38 \times 10^6\,m$

$R_f = 6.38 \times 10^6\,m$

$E_i = E_f$

$$\cancel{\frac{1}{2}mv_i^2}^{\,0} + \left(-\frac{GM\cancel{m}}{R_i}\right) = \cancel{\frac{1}{2}}\cancel{m}v_f^2 + \left(-\frac{GM\cancel{m}}{R_f}\right)$$

→ rearrange for v_f

$$v_f = \sqrt{\left(-\frac{GM\cancel{m}}{R_i} + \frac{GM\cancel{m}}{R_f}\right) \times 2}$$

$$= \sqrt{2\left(-\frac{6.67\times10^{-11}(5.98\times10^{24})}{3.5\times10^{5} + 6.38\times10^{6}} + \frac{6.67\times10^{-11}(5.98\times10^{24})}{6.38\times10^{6}}\right)}$$

$$= \boxed{2600\,m/s}$$

a) $v_i = 0\ m/s^2$ $\quad v_f = ?$

$R_i = \infty$ $\quad R_f = 6.38\times 10^{6}\ m$

$E_i = E_f$

$$\cancel{\frac{1}{2} m v_i^2}^{\,0} + \cancel{\left(-\frac{GMm}{R_i}\right)}^{\,0} = \frac{1}{2} m v_f^2 + \left(-\frac{GMm}{R_f}\right)$$

$R_i \rightarrow \infty$

$$0 = \frac{1}{2}\cancel{m} v_f^2 - \frac{GM\cancel{m}}{R_f}$$

$$\frac{GM}{R_f} = \frac{1}{2} v_f^2$$

$$v_f = \sqrt{\frac{2GM}{R_f}}$$

$$= \sqrt{\frac{2(6.67\times 10^{-11})(5.98\times 10^{24})}{6.38\times 10^{6}\ m}} = \boxed{11\,000\ m/s}$$