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5.3 Gravitation and satellite orbits

Recall from last year about Newton's Universal Law of Gravitation:

Where m_1 and m_2 are ____________________

r is the ________________________________

G is the ________________________________

____

F_G is the attractive force between mass 1 and mass 2 that is directly proportional to their masses and inversely proportional to their distance apart.

We can also quantify the **gravitational field strength** as the *force per unit mass* acting on a mass placed at a particular location. Using Newton's Universal Law of Gravitation:

$$g = \frac{GM}{r^2}$$

Try googling the Earth's mass and radius to calculate the Earth's gravity on its surface!

Some useful numbers:

Earth's radius: 6.38×10^6 m

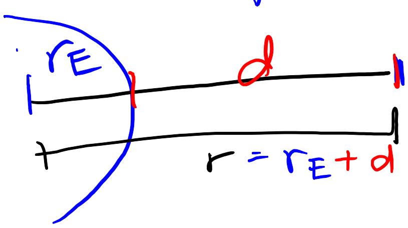
Earth's mass: 5.97×10^{24} kg

N/kg m/s^2

field

Ex.1: Calculate the distance you'd have to be **above the Earth's surface** for the gravitational strength to be ½ that of gravity? ~~(ANS: 2.6×10^6 m)~~

½ of 9.81 m/s^2



$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(r_E+d)^2}$$

$$\Rightarrow g(r_E+d)^2 = GM_E \quad \frac{g(r_E+d)^2}{g} = \frac{GM_E}{g}$$

$$\Rightarrow (r_E+d) = \sqrt{\frac{GM_E}{g}}$$

$$\cancel{r_E} + d = \sqrt{\frac{GM_E}{g}} - r_E$$

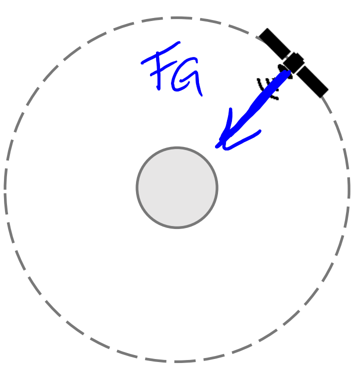
$$d = \sqrt{\frac{GM_E}{g}} - r_E = \sqrt{\frac{6.67E{-}11(5.97E24)}{\frac{1}{2}(9.81)}} - 6.38E6 = \boxed{2.6 \times 10^6\,m}$$

How do we apply this to satellites in orbit?

Imagine you're trying to shoot something horizontally off the cliff. Of course, it will fall in a parabolic manner. What would happen if you could shoot this object with VERY HIGH speed? Consider the following demonstration: http://www.angrybirdsgames.com/games/angry-birds-space

With enough _______________________, an object can be placed into orbit at a certain distance above the Earth's surface. An object in orbit only experiences the force

____.



Compare with free-fall:

In both cases, a person inside would feel

______________________________ because the person

is ____________________ towards the

_________________ of the Earth due to the gravitational

force. You're always falling, but you're moving so fast that you keep missing the ground.

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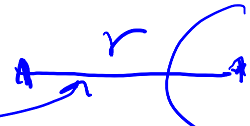


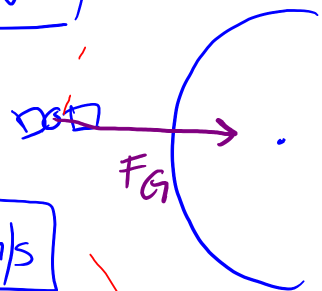
Ex. 2: A satellite weighs 9.00×10^3 N on the Earth's surface. How much does it weigh if its mass is tripled and its orbital radius is doubled? *Note: orbital radius is the total distance between the orbiting object and the larger object's center of mass.* (ANS: 6750 N)

weight

$$F_G = \frac{G M_E (3m_s)}{(2r)^2} = \frac{3}{4}\boxed{\frac{G M_E m_s}{r^2}}_{F_G} = \frac{3}{4}(9.00\times10^3 N) = \boxed{6750 N}$$

Ex. 3: A 4500 kg Earth satellite has an orbital radius of 8.50×10^7 m.
a) At what speed does it travel? (ANS: 2200 m/s)
b) What is its orbital period? (ANS: 250000s)

$$v = \frac{2\pi r}{T}$$



a) $F_{net} = F_G = m a_c \Rightarrow \frac{G M_E \cancel{m_s}}{r^{\cancel{2}}} = \cancel{m_s}\frac{v^2}{r}$

$$\sqrt{\frac{GM_E}{\cancel{r}}} = \sqrt{\frac{v^2}{\cancel{r}}} \Rightarrow v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{6.67E{-11}(5.97E24)}{8.50\times10^7 m}} = \boxed{2200 m/s}$$

b) $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(8.50\times10^7 m)}{2164\ldots m/s} = 246750 s = \boxed{250000 s}$

Consider the following situations. If Satellite A is identical to Satellite B, which one would be traveling faster? Why?
a) Satellite A orbits the Earth at twice the orbital radius of Satellite B.
b) Satellite A orbits the Sun at the same orbital radius that Satellite B orbits the Earth.

What does the orbital period depend on?

Most satellites that we use are positioned above our geographic location such as communications and weather satellites. However, our Earth rotates around its axis once every 24 hours. That means that these satellites must have an orbital period of ______24 hrs______.

These orbits are called **geosynchronous orbits** or **geostationary orbits**.

Ex. 4: Find the orbital radius of a satellite that is geosynchronous above the Earth's equator. What is the speed of this satellite? (ANS: $r = 4.2 \times 10^7$ m; $v = 3.1 \times 10^3$ m/s)

$$v = \frac{2\pi r}{T} \qquad T = 24 hrs.$$

$$F_{net} = F_G = m a_c \Rightarrow \frac{G M_E m_s}{r^2} = \frac{m v^2}{r} \Rightarrow \frac{G M_E \cancel{m_s}}{r^2} = \frac{\cancel{m_s} 4\pi^2 r}{T^2} \times r^2$$

$$\Rightarrow GM_E = \frac{4\pi^2 r^3}{T^2} \Rightarrow r = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} \qquad v = \frac{2\pi r}{T}$$

$$= \sqrt[3]{\frac{6.67E{-11}(5.97E24)(24\times3600s)^2}{4\pi^2}} = \boxed{4.2\times10^7 m}$$

$$v = \frac{2\pi(4.2\ldots\times10^7 m)}{24\times3600} = \boxed{3.1\times10^3 m/s}$$

HW: Worksheet 5.3 ALL