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5.2 Circular Motion: Vertical circles and banked curves

After watching the demo, let's answer a few questions:

(Alternatively, there's a video to show something similar (Spill Not): https://www.youtube.com/watch?v=Atg3382eLMU

F_N, F_g

1) Why did the liquid stay in its container? Fnet directed towards the center of the circle

2) What happens if we spin the container slower? water spills

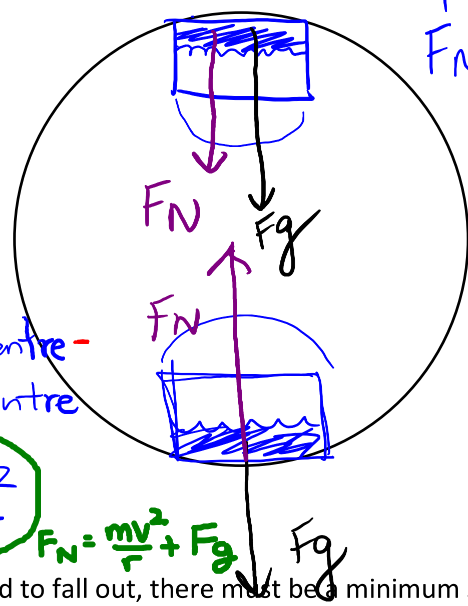
3) What is the force acting on the liquid at the top of the container that allows it to stay? $F_N + F_g$

4) If YOU were the liquid in the container, at which point during the ride would you feel the heaviest? The lightest?

heaviest - bottom, lightest - top.

(Check this out: Science of Stupid Season 1 Episode 6 (7:50): https://www.youtube.com/watch?v=hnASMjw08z8

5) Let's draw a free-body diagram of this



Top:

$$F_{net} = F_N + F_g = \frac{mv^2}{r} \quad (= ma_c)$$

$$F_N = \frac{mv^2}{r} - F_g$$

Bottom:

Fnet = Forces towards centre - Forces away from centre

$$= F_N - F_g = \frac{mv^2}{r} \quad (= ma_c)$$

$$F_N = \frac{mv^2}{r} + F_g$$

If spinning too slow will cause the liquid to fall out, there must be a minimum speed we need for the liquid to clear the top of the circle. Let's watch a video to understand the conditions of this minimum speed:

Tony Hawk's Loop of Death: https://www.youtube.com/watch?v=Y8dIIRCBuOI

6) For a skateboarder who barely makes it through the top of the loop. how much normal force was acting on the skateboarder at that point? $F_N = 0$

7) If that's the case, how heavy should he/she feel? weightless

Let's look at an example:

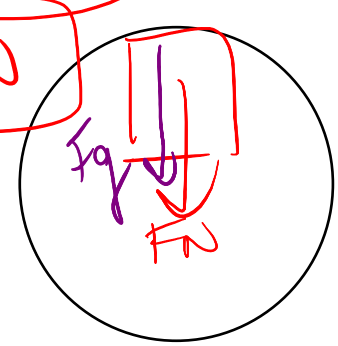
Ex. 1: A skateboarder is trying to clear a loop de loop with a radius of 3.0m. What is the minimum speed of the object at the top of the loop to clear the top? (ANS: 5.4m/s)

$$\boxed{F_N = 0N}$$

$$F_{net} = F_g + \cancel{F_N}^{\,0} = ma_c$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{gr} = \sqrt{9.81(3.0)} = \boxed{5.4 m/s}$$



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Here is another example:

Ex. 2: A 1.7 kg object is swung from the end of a 0.60m string in a vertical circle. If the time of one revolution is 1.1s, what is the tension in the string:

T

a) at the top? (ANS: 17N)

F_{net} = Forces towards centre − Forces away from centre

$= F_T + F_g$

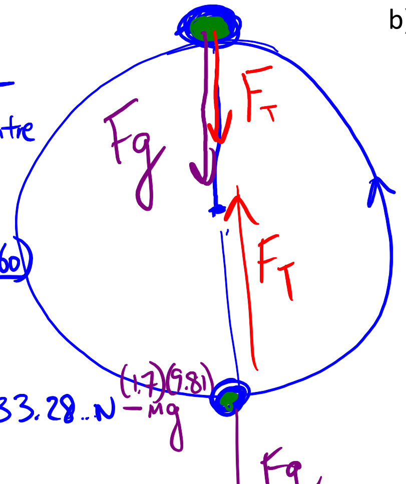
$F_{net} = ma_c = \frac{m4\pi^2 r}{T^2} = \frac{1.7(4)\pi^2(0.60)}{1.1^2}$

$= 33.28...N$

$33.28..N = F_T + F_g$

$F_T = 33.28..N - F_g = 33.28..N - mg$ (1.7)(9.81)

$= \boxed{17N}$



b) at the bottom? (ANS: 50.N)

$F_{net} = F_T - F_g = ma_c$

$F_T = ma_c + mg$

$= \frac{m4\pi^2 r}{T^2} + mg$

$= m\left(\frac{4\pi^2 r}{T^2} + g\right) = \boxed{50.N}$

Banked Curves

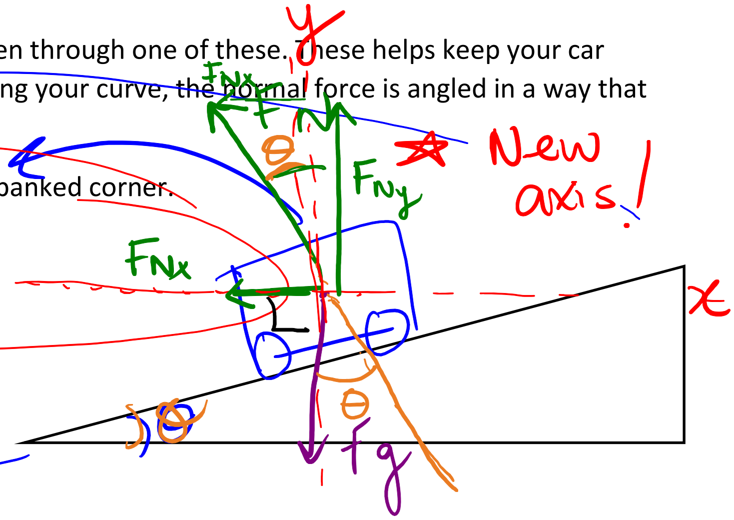
When traveling at high speeds on a highway, you might have driven through one of these. These helps keep your car moving in a circle that friction alone may not be enough. By banking your curve, the normal force is angled in a way that some of it helps keep your car moving in that circle.

Consider a car traveling at a constant speed around a frictionless banked corner.

CAREFUL!

$F_N \neq F_g \cos\theta$

F_N is actually > than F_g because the car is not accelerating **downwards** but **sideways** towards the center of the circle.



Ex. 3: Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22m/s if its radius is 475m. (ANS: 6.0°)

x: $F_{net} = F_{Nx} = ma_c$

$= F_N \sin\theta = ma_c$

y: $\vec{a}_y = 0 \therefore F_{Ny} = F_g$

$F_N\cos\theta = mg \Rightarrow F_N = \frac{mg}{\cos\theta}$

substitute F_N back into my x eqns

$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a_c$ $\frac{\sin\theta}{\cos\theta} = \tan\theta$!

$\tan\theta\, g = a_c \Rightarrow \tan\theta = \frac{a_c}{g} = \frac{v^2/r}{g}$

$\theta = \tan^{-1}\left(\frac{v^2/r}{g}\right) = \tan^{-1}\left(\frac{22^2/475}{9.81}\right) = \boxed{6.0°}$

HW: Worksheet 5.2 #1-8

Ex. 4: A 0.25kg toy plane is attached to a string so that it flies in a horizontal circle with a radius of 0.80m. The string makes a 28° to the vertical. What is its period of rotation? (ANS: 2.5s)

y: $a_y = 0$ $F_g = F_{Ty} = F_T\cos 28$

$F_T = \frac{F_g}{\cos 28}$

x: $F_{net} = ma_c$

$F_{Tx} = F_T \sin 28 = ma_c = \frac{m4\pi^2 r}{T^2}$

$\frac{F_g}{\cos 28}\sin 28 = \frac{m4\pi^2 r}{T^2}$ ($F_g = mg$)

$\boxed{T = \sqrt{\frac{m4\pi^2 r\cos 28}{mg\sin 28}}}$

$T = \sqrt{\frac{4\pi^2(0.80m)\cos 28}{9.81\sin 28}} = \boxed{2.5s}$